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State Dependent Bulk Service Queue with Delayed Vacations

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ABSTRACT. This investigation is concerned with the study of M/M (a, d, b)/1 queueing system with multiple vacations wherein finding (a-1) customers waiting in the queue, the server will wait in the system for some time (changeover time), instead of going for vacation immediately. The server starts service on finding an arrival during the changeover time otherwise the server will go for a vacation. There is provision to entertain late arriving customers in the ongoing service batch if the size of the batch being served is less than a threshold integer d $(a \le d \le b)$. The steady state queue size distribution is established which is further employed to determine average queue length. In order to match our investigation with the earlier existing results, we discuss some particular cases. Tables and graphs to demonstrate the effect of traffic intensity on expected queue length for both vacation and non-vacation models are provided.

KEY WORDS: Bulk queue, Delayed vacations, Changeover time, Expected queue length.

1. Introduction

The last two decades have witnessed a tremendous growth in the applications of bulk queues to many congestion situations. The primary reason for this is that bulk queueing models are often encountered in real life systems such as transportation systems, computer system, telecommunication, airline scheduling as well as industrial processes such as production/ inventory systems etc. Powell and Humblet (1986) applied general control strategy for bulk service queue. Jayaraman *et al.* (1994) proposed arrival rate dependent on server-breakdown for general bulk service queue. Willmot and Drekic (2001) proposed transient analysis for the $M^x / M / \infty$ queue. Dshalalow (2001) described briefly D-policy for bulk queueing system.

Borthakur (1975) obtained the laplace transform of the pdf of the busy period for the $M/M^{(a,b)}$ /1 model and calculated busy-period distribution for $M^X/G^{(a,b)}/1$ queueing system. Various operational characteristics for $E^k/M^{(a,b)}/1$ queueing system were derived by Easton and Chaudhry (1982). Kambo and Chaudhry (1984) considered $E^k/M^{(a,b)}/1$ system to obtain busy period distribution and control strategy of bulk service queueing model. Solanki (1998) gave transient analysis for $E_k/M^{(a, b)}/1/N$ queueing system.

Queueing models of bulk service with accessible and non-accessible batches have been investigated by several authors. The concept of non-accessibility while receiving service, has been studied by Medhi (1975, 1979), Weiss (1979). Sivasamy (1990) analyzed a bulk service queue with accessible and non-accessible batches. Jain and Sharma (1991) obtained results for average queue length and waiting time distribution for state dependent bulk service system with accessible and non-accessible batches. Recently, Jain and Sharma (2002) established the expression for average queue length for state dependent $M^r/M^{(a, d, b)}/1$ queue with accessible and non-accessible batches.

During the last three decades, considerable attention was paid to analyse the queueing system with vacation. Many researchers have analyzed vacation queueing problems for both Markovian and non-markovian queueing models. The markovian models may be more appropriate in practice and may have certain advantages. For a detailed comprehensive survey on queueing systems with server vacations and priority systems, one can refer to Doshi (1986) and Takagi (1991). Fuhrmann and Cooper (1985) analyzed stochastic decomposition in an M/G/1 queue with generalized vacation. Kella (1989) described threshold policy for the M/G/1 queue with server vacations. Choudhary (1995) considered M/G/1 queueing system with vacation and setup time. Boxma and Yechaiali (1997) introduced multiple types of feedback and gated vacations for M/G/1 queue. Li and Zhu (1997) analyzed M(n)/G/1/N queue with generalized vacations. Gray *et al.* (2000) investigated a vacation queueing model with service breakdowns. Lee *et al.* (2001) gave an analysis of multiple-class vacation queues with individual threshold.

The inclusion of vacation into bulk queueing models makes the models more feasible to the practical situations. Some researchers have also paid their attention towards this direction. Using matrix geometric method, Nadarajan and Subramaniyam (1984) analyzed $M/M^{(a,b)}/1$ queueing system with server's vacation. Altman and Nain (1996) determined the optimal threshold and the optimal long-run average cost in he M/M/1 queue with repeated vacations. Li and Zhu (1996) provided an explicit formula for the Laplace transform of the additional delay for M/G/1 queues with delayed vacations and exhaustive service discipline. Hassan and Atiquzzaman (1997) developed a new delayed vacation model to facilitate the performance indices of SVCCs in the ATM networks. Reddy and Anitha (1998) determined the stationary distribution of the number of customers in queue and waiting time distribution of an arriving customer for M/M (a, b)/1 queueing system with multiple vacations and changeover time. Xiuli and Zhao (1998) derived steady state probabilities and made comparison of station vacation and server vacation models based on numerical observations. Reddy *et al.* (1998) presented analysis of a bulk queue having the assumptions of N-policy, multiple vacations and setup times. Choudhury (2001) obtained the expected delay in services due to vacation period for M/M/1 queueing system under N-policy and exponential setup time.

In this investigation, we study an M/M(a,d,b)/1 queueing system with multiple vacations and changeover time having state dependent service rate. The remaining part of the paper is organized as follows. Section 2 provides the description of the model. Section 3 presents the mathematical analysis of the system. In section 4, we derive the expected queue length. In section 5, some particular cases have been presented, the results for which tally with earlier existing results. Numerical results are facilitated to analyze the effect of system parameters on the expected queue length in section 6. In section 7, some concluding remarks and notable features of investigation done are highlighted.

2. Description of the System

Consider $M/M^{(a,d,b)}/1$ queueing system with repeated delayed vacations and changeover time. The basic features of the model are described as follows:

- The customers join the system singly according to Poisson process with arrival rate λ.
- Service times follow an exponential distribution with mean service rate μ_n.

  where $$\mu_n = \begin{cases} \mu , & n = 0 \\ \mu_1 , & n > 0 \end{cases}$$
- The customers are served in batches with a quorum level "a" and quota capacity "b".
- If there are j customers $(0 \le j \le a-2)$ in the queue, the server will go for a vacation, which is exponentially distributed with parameter β.

- If the server finds a-1 customers in the queue either at a service completion epoch or at a vacation termination point, he will wait for some more time in the system which is called changeover time.
- The changeover time is exponentially distributed with parameter α.
- When an arrival occurs during this changeover time, the server will start service immediately otherwise at the end of the changeover time, the server will go for a vacation.
- On returning to the system after a vacation, if the server finds $0 \le j \le a-2$ customers in the queue, the server will go for another vacation and so on until the server, on his arrival finds at least a-1 customers in the queue.
- Vacations, service times and inter-arrival times are mutually independent and identically distributed nonnegative random variables.
- The late arriving customers are allowed to join the batch (without affecting the service time) in case of ongoing service as long as the number of units in that batch is less than 'd'.

3. Mathematical Analysis

We formulate the process as a continuous time Markov chain with the state space $\{(i,j)/i \ge 0, j = 0,1\} U \{(a-1,2)\}$ where i gives the queue size and j represents the state of the server.

The process is said to be in the state (i, 0) if there are i customers waiting in the queue and the server is away for vacation. The process remains in the state (i, 1) if there are i customers waiting in the queue and the server is busy. The process is said to be in the state (a-1, 2) if there are a-1 customers waiting in the queue and the sever is waiting in the system.

We define $p_{i,j}(t) = \Pr ob.\{The\ system\ is\ in\ state(i,j)\ at\ time\, t\}$

Let us assume that the steady state probabilities $p_{i,j} = \underset{t\to\infty}{Lim}\, p_{i,j}(t)$ exists

We obtain the following system of equations

$$\lambda p_{0,0} = \mu p_{0,1} \tag{1}$$

$$\lambda p_{i,0} = \lambda\ p_{i-1,0} + \mu p_{i,1}, \qquad 1 \le i \le a-2 \tag{2}$$

$$(\lambda + \beta) p_{a-1,0} = \lambda p_{a-2,0} + \alpha p_{a-1,2} \tag{3}$$

$$(\lambda + \mu_1) p_{i,0} = \lambda p_{i-1,0} + \mu p_{i,1} \qquad a \le i \le d-1 \tag{4}$$

$$(\lambda + \beta)p_{i,0} = \lambda_1 p_{i-1,0} \ , \qquad i \geq a \tag{5}$$

$$(\lambda + \mu_1)p_{0,1} = \lambda p_{a-1,2} + \mu \sum_{s=d}^{b} p_{s,1} + \beta \sum_{s=d}^{b} p_{s,0} \tag{6}$$

$$(\lambda + \mu_1)p_{i,1} = \lambda p_{i-1,1} + \beta \, p_{i+b,0} + \mu_1 p_{i+b,1} \ , \qquad i \geq 1 \tag{7}$$

$$(\lambda + \alpha)p_{a-1,2} = \mu p_{a-1,1} + \beta \, p_{a-1,0} \tag{8}$$

3.1 Steady State Probabilities

Equation (7) can be written as

$$\left[\mu E^{b+1} - (\lambda + \mu)E + \lambda\right]p_{i,1} = \beta p_{i+b+1} \qquad i \geq 1 \tag{8.1}$$

Using the forward shifting operator E defined by $Ep_{i,1} = p_{i+1,1}$, the characteristic equation of (8.1) becomes

$$\mu z^{b+1} - (\lambda + \mu)z + \lambda = 0 \tag{9}$$

When $\frac{\lambda}{b\mu_1} < 1$, Rouche's theorem yields that there exists only one real root of this equation inside the circle $|z| = 1$. Assuming the stability condition $\rho = \frac{\lambda}{b\mu_1} < 1$, if r is the root of the above characteristic equation with $|r| < 1$, then $b\rho = \frac{\lambda}{\mu_1} = \frac{r(1 - r^b)}{(1 - r)} = r + r^2 + r^3 + ...r^b$ (10)

Using the concept that $p_{i,1} < 1$, the solution of equation (8.1) is

$$p_{i,1} = \left(Ar^i - B\theta^i\right)p_{a-1,0} \ , \quad i \geq 0 \tag{11}$$

where A is an arbitrary constant and

$$B = \frac{\beta\theta^{b-a+1}}{\mu\theta^b - \mu + \beta} \tag{12}$$

Solving equation (8) for $p_{a-1,2}$ and using equation (11), we get

$$p_{a-1,2} = \frac{1}{\lambda + \alpha}\left[A\mu_1 r^{a-1} - B\mu_1\theta^{a-1} + \beta\right]p_{a-1,0} \tag{13}$$

Summing equation (2) over all relevant i and adding equation (1), we get

$$\lambda\sum_{i=0}^{a-2} p_{i,0} = \lambda\sum_{i=0}^{a-3} p_{i,0} + \mu\sum_{i=0}^{a-2} p_{i,1}$$

It follows that

$$p_{a-2,0} = \frac{\mu_1}{\lambda}\left[\frac{A\left(1-r^{a-1}\right)}{(1-r)} - \frac{B\left(1-\theta^{a-1}\right)}{(1-\theta)}\right]p_{a-1,0} \qquad (14)$$

Solving equation (2) recursively, we get

$$p_{i,0} = \frac{\mu}{\lambda}\left[(A-B) + \alpha\frac{Ar\left(1-r^{i}\right)}{(1-r)} - \alpha\frac{B\theta\left(1-\theta^{i}\right)}{(1-\theta)}\right]p_{a-1,0}, \qquad 1 \le i \le a-2 \qquad (15)$$

We solve equation (5) recursively and get

$$p_{i,0} = \frac{1}{\gamma^{i+1-a}}\left[1 + \frac{\mu_1}{\lambda}\left\{Ar^{a}\left(\frac{1-(\gamma r)^{i+1-a}}{1-\gamma r}\right) - B\theta^{a}\left(\frac{1-(\gamma\theta)^{i+1-a}}{1-\gamma\theta}\right)\right\}\right]p_{a-1,0},$$

$$a \le i \le d-1 \qquad (16)$$

where $\gamma = \dfrac{\lambda}{\lambda+\mu_1}$

From equation (5), we get

$$p_{i,0} = \theta^{i+1-a}p_{a-1,0} \qquad i \ge a-1 \qquad (17)$$

where $\theta = \dfrac{\lambda}{\lambda+\beta}$

Equations (11), (13) and (15-17) give all the steady state probabilities $p_{i,j}$ in terms of $p_{a-1,0.}$ Using the normalizing condition

$$\sum_{i=0}^{\infty} p_{i,1} + \sum_{i=0}^{a-1} p_{i,0} + \sum_{i=a}^{d-1} p_{i,0} + p_{a-1,2} = 1 \qquad (18)$$

we get

$$p_{a-1,0} = \left[AH(r) - BH(\theta) + \frac{\mu}{\lambda}(A-B)(a-1) + \left(\frac{\gamma^{a-d}-1}{1-\gamma}\right) + \frac{\beta}{\lambda+\alpha}\right]^{-1} \qquad (19)$$

where

$$H(y)=\frac{1}{1-y}+\frac{\mu\eta y}{\lambda(1-y)}\left\{(a-1)-\frac{\left(1-y^{a}\right)}{(1-y)}\right\}-\frac{\mu_1}{\lambda}\left(\frac{\gamma^{a-d}-1}{1-\gamma}\right)\left(\frac{\left(1-y^{d-a}\right)}{(1-\gamma y)(1-y)}\right)y^{a+1}$$
$$+\frac{\mu_1 y^{a-1}}{\lambda+\alpha} \tag{20}$$

Substituting the probabilities $p_{a-1,0}$, $p_{a-2,0}$ and $p_{a-1,2}$ in equation (3), we get

$$A=\frac{\left[\frac{\lambda(\lambda+\alpha+\beta)}{\lambda+\alpha}+\frac{B\mu}{(1-\theta)}\left\{1-\frac{R^{a-1}(\lambda+\alpha R)}{(\lambda+\alpha)}\right\}\right]}{\left\{\frac{\mu\left(1-r^{a-1}\right)}{(1-r)}+\frac{\alpha\mu r^{a-1}}{\lambda+\alpha}\right\}} \tag{21}$$

4. Expected Queue Length

The expected queue length L_s in steady state is given by

$$L_s=\sum_{i=0}^{d-1} i p_{i,0}+\sum_{i=0}^{\infty} i p_{i,1}+(a-1)p_{a-1,2} \tag{22}$$

Substituting for $p_{i,1}, p_{a-1,2}, p_{i,0}(1\le i\le a-2)$ and $p_{i,0}(a\le i\le d-1)$ from equations (11), (13), (15) and (16) respectively and simplifying, we get

$$L_s=\left[AF(r)-BF(\theta)+\frac{a(a-1)(A-B)}{2\lambda}+\right.$$
$$\left.\gamma^{a-d}\left\{\frac{(d-1)-a\gamma^{d-a}}{1-\gamma}-\frac{\left(\gamma-\gamma^{d-a}\right)}{(1-\gamma)^2}\right\}+\frac{\beta(a-1)}{(\lambda+\alpha)}\right]p_{a-1,0} \tag{23}$$

where

$$F(x)=\frac{x}{(1-x)^2}+\left(\frac{\mu\eta x}{\lambda(1-x)}\right)\left\{\frac{a(a-1)}{2}-\frac{\left(x-x^{a}\right)}{(1-x)^2}+\frac{(a-1)x^{a}}{(1-x)}\right\}-$$
$$\frac{\mu_1\gamma^{a-d}x^{a+1}}{\lambda(1-\gamma x)}\left\{\frac{a-(d-1)x^{d-a}}{(1-x)}+\frac{\left(x-x^{d-a}\right)}{(1-x)^2}\right\} \tag{24}$$

5. Particular Cases

In this section, we deduce some particular cases which tally with earlier existing results.

Case 1: When $\mu_1 = \mu$ i. e., the service rate of the server is independent of the number of customers present in the system, our model reduces to M/M (a, d, b)/1 queueing model with repeated delayed vacations and changeover time.

Case 2: When $d = b$, we get the expected queue length for general bulk service M/M (a, b)/1 queueing system with repeated server vacation and change over time with state dependent service rate as

$$L_q = \left[AF(r) - BF(\theta) + \frac{\mu}{\lambda}(A-B)(a-1) + \frac{(a-1)\beta}{\lambda+\alpha} \right] p_{a-1,0} \tag{25}$$

$$F(x) = \frac{x}{(1-x)^2} + \frac{(a-1)(a-2)\gamma x}{2(1-x)} - \frac{\gamma x^2\left(1-x^{a-2}\right)}{(1-x)^3} + \frac{\gamma(a-2)x^a}{(1-x)^2} + \frac{(a-1)\mu_1 x^{a-1}}{\lambda+\alpha} \tag{26}$$

Case 3: When $\mu_1 = \mu$ and $d = b$, then our results tally with the general bulk service queueing system M/M (a, b)/1 with repeated server vacation and changeover time. [cf. Reddy and Anitha (1998)].
In this case, the explicit expression for average queue length reduces to

$$L_q = \left[AF(r) - BF(\theta) + \frac{\theta + a - 1}{1-\theta} + \frac{\vartheta^2}{(1-\theta)^2} + \frac{(a-1)\beta}{\lambda+\alpha} \right] p_{a-1,0} \tag{27}$$

where

$$F(x) = \frac{\mu(a-2)x^a}{\lambda(1-x)^2} + \frac{\mu(a-1)x^{a-1}}{\lambda+\alpha} - \frac{\mu x^2\left(1-x^{a-2}\right)}{\lambda(1-x)^3} + \frac{x}{(1-x)^2} + \frac{\mu(a-1)(a-2)}{2\lambda(1-x)} \tag{28}$$

Case 4: When $\mu_1 = \mu$, $d = b$ and $\alpha \to \infty$, i.e. without assumptions of state dependent service rate and changeover time, the model becomes M/M (a, b)/1 model with repeated vacation.

Case 5: When $\mu_1 = \mu$, $d = b$, $\alpha \to \infty$ and $\beta \to \infty$, the results coincide with the corresponding results of M/M (a, b)/1 model discussed by Medhi (1979).

6. Sensitivity Analysis

Sensitivity analysis is carried out to demonstrate the effect of various parameters on the system performance. Numerical results are tabulated and demonstrated in Table 1 and Figures 1-3 respectively. Table 1 presents the comparison between expected queue length for vacation and non-vacation models with state dependent and constant service rates by varying d for different sets of (a, b). We observe that the expected queue length L_s increases with d.

Table 1: Comparison of Expected queue length for b=45, α=10, β=2.

d	***(a, b)***	***L_s***			
		State dependent service rate		***Constant service rate***	
		Vacation	***Non-vacation***	***Vacation***	***Non-vacation***
d=20	(8,30)	11.518	13.955	11.422	11.506
	(10,32)	13.377	15.191	12.359	13.370
	(12,34)	15.244	16.621	13.474	15.181
	(14,36)	16.971	18.149	14.679	16.968
	(16,38)	18.745	19.738	15.936	18.743
	(18,40)	20.508	21.363	17.225	20.507
d=25	(8,30)	11.542	13.975	11.444	11.530
	(10,32)	13.410	15.214	12.384	13.403
	(12,34)	15.447	16.648	13.504	15.227
	(14,36)	17.035	18.185	14.717	17.032
	(16,38)	18.833	19.784	15.986	18.831
	(18,40)	20.631	21.425	17.291	20.629
d=30	(8,30)	11.550	13.981	11.451	11.538
	(10,32)	13.421	15.221	12.392	13.414
	(12,34)	15.857	16.657	13.513	15.242
	(14,36)	17.056	18.196	14.729	17.053
	(16,38)	18.862	19.799	16.002	18.861
	(18,40)	20.672	21.445	17.313	20.670

Figure 1 displays the effect of traffic intensity on expected queue length L_s with variation for non-vacation and vacation models. It is noted that the expected queue length L_s increases with the increase in batch size a. The expected queue length L_s for non-vacation model is higher than that of vacation model. Figure 2 exhibits the comparison between expected queue length L_s for models with state dependent and constant service rates for different values of a. It is easily observed that the expected queue length L_s increases with ρ_1 and a. Light traffic does not affect L_s significantly whereas significant improvement is noticed for heavy traffic. Figure 3 shows a comparison between expected queue length for model with constant and state dependent service rate for two sets of (a, b). We observe that expected queue length for state dependent service rate is higher than that of the constant service rate.

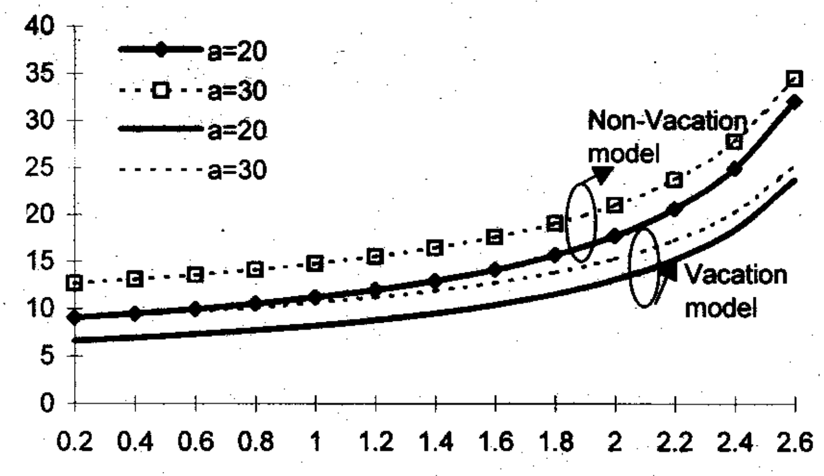

Fig. 1: Expected queue length (Ls) for vacation and non-vacation model (β=10, b=40, d=35)

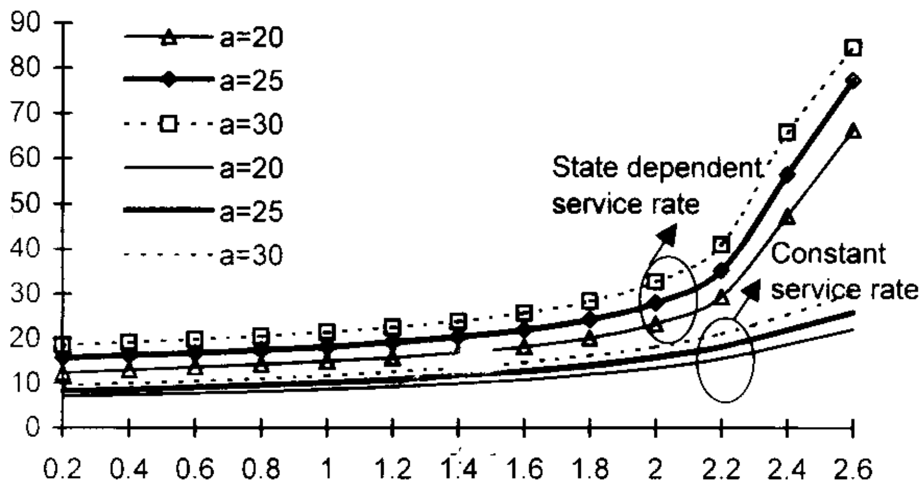

Fig. 2: Expected queue length (Ls) for model with state dependent and constant service rates (α=2, β=10, d=35)

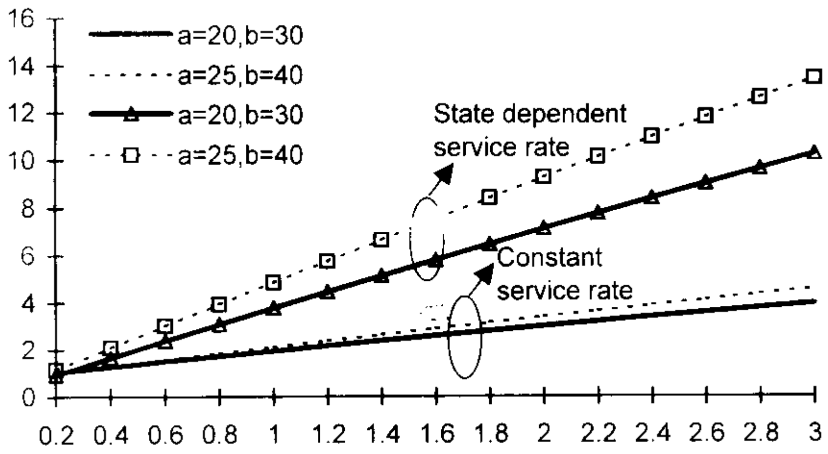

Fig. 2: Expected queue length (Ls) for model with state dependent and constant service rates (α=10, β=10, d=35)

Finally, we can conclude that:

- The expected queue length L_s for state dependent service rate is higher than that of constant service rate.
- The expected queue length L_s for state dependent and constant service rates increases nearly linear for light traffic whereas it increases sharply for heavy traffic.
- The expected queue length L_s for non-vacation model is higher than that of vacation model.

7. Discussion

We have analyzed general bulk service queueing model with repeated delayed vacations and state dependent service rate. The notable feature of present investigation is to allow late arriving customers into servicing facility without affecting the service if the size of the batch served is less than d $(a \le d \le b)$. Such situations can be visualized in many real life transportation systems such as in shuttle bus service, taxi, express elevators, tour guides. The incorporation of state dependent service rate makes our model more feasible from application point of view in many real time systems. The explicit expressions for expected queue length may be helpful in setting traffic management strategies based on performance indices.

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طوابير الخدمة الضخمة المتأثرة بالحالة ذات الإجازات المؤخرة

مادهي جان و بونام سنف

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المستخلص: يتعلق هذا البحث بدراسة نظم طوابير ذات الإجازات المتعددة والتي يكون فيها عدد من العملاء المنتظرين في الطابور وينتظر الخادم فيها لبعض الوقت (وقت التغيير) بدلاً من إعطاء الإجازة مباشرة. ويبدأ الخادم الخدمة عند وصول طلب خلال وقت التغيير وإلا فإنه يعطي الإجازة. ويكون هناك إمكانية للتعامل مع الطلبات المتأخرة ضمن المجموعة الحالية إذا كان حجم المجموعة أقل من حد معين، ويتم تحديد درجة توزيع طول الطابور في الحالة المستقرة والذي يستخدم بالتالي في تحديد متوسط طول الطابور، ولمقارنة نتائج هذه الدراسة مع بعض النتائج السابقة يتم مناقشة بعض الحالات الخاصة. كما يتم عرض بعض الجداول والرسومات التي توضح تأثير كثافة السير في الطابور على طول الطابور المتوقع لحالات الإجازات وبدون إجازات.