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LIMITATIONS OF ADOMIAN DECOMPOSITION AND HOMOTOPY METHODS

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Abstract: It is usual to resort to Adomian decomposition method or to homotopy analysis method for the solution of nonlinear problem. We show by considering a simple example that, in the absence of an asymptotic analysis, these methods fail to provide any significant information beyond a finite interval.

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Key Words: Adomian decomposition method, homotopy analysis, nonlinear problem

1. Introduction

If a problem contains a small parameter, the perturbation method prove to be very useful in the solution [5]. If no such parameter exists, Adomian introduce his decomposition method [1, 2, 3] which has been applied to solve some nonlinear problems, see for example [7, 8]. Liao introduce a technique, called the homotopy analysis method which contains the Adomian decomposition method as a special case [4]. In this method there are several free parameters which can be chosen so that the solution becomes valid over a large domain. However if the initial approximation is not properly chosen both these methods yield power series which converge only for a finite interval and fail to provide any information concerning large values of the independent variable. In view of this, one may conclude that claims of these two methods are sometimes exaggerated.

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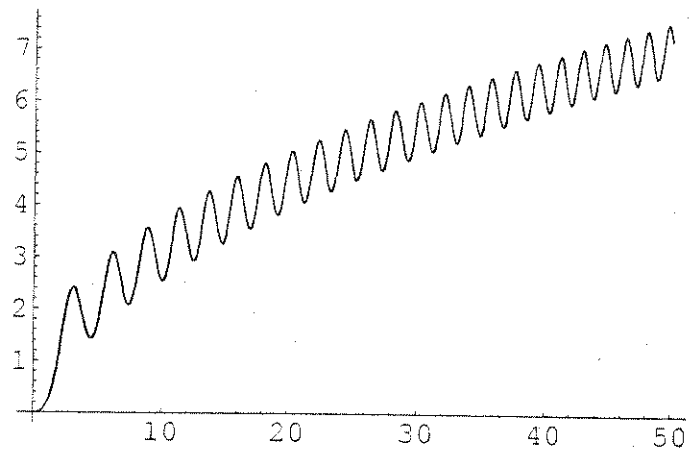

Figure 1: Numerical solution of equation (1)

Compare (7) with (9). We get

$$1-2s=0\,,\quad s^2-r^2\alpha^2=0\,,\quad 2r=\frac{5}{2}\,,\quad a^2r^2=2\,.$$

Therefore $s=\frac{1}{2}$, $r=\frac{5}{4}$, $\alpha=\frac{2}{5}$, $a=\frac{4\sqrt{2}}{5}$ and the general solution of (6) is

$$u=\sqrt{x}[c_1J_{\frac{2}{5}}(\frac{4\sqrt{2}}{5}x^{\frac{5}{4}})+c_2Y_{\frac{2}{5}}(\frac{4\sqrt{2}}{5}x^{\frac{5}{4}})]\,.$$

Since the Bessel functions are oscillatory with decreasing amplitude such that

$$J_\alpha(x)\to 0 \qquad \text{as } x\to\infty\,,$$
$$\text{and}\quad Y_\alpha(x)\to 0 \qquad \text{as } x\to\infty\,.$$

our assumption that $|u|\ll\sqrt{x}$ is justified for large x.

From (3) we see that the solution of equation (1), for large x, will be small oscillations superimposed on the parabola $y=\sqrt{x}$.

In Figure 1 we present the numerical solution of the problem which confirms our qualitative analysis. In Figure 2 we present the solution along with the curve $y=\sqrt{x}$.

3. Solution by Adomian Method

If we solve the problem by Adomian decomposition method, we get the following solution

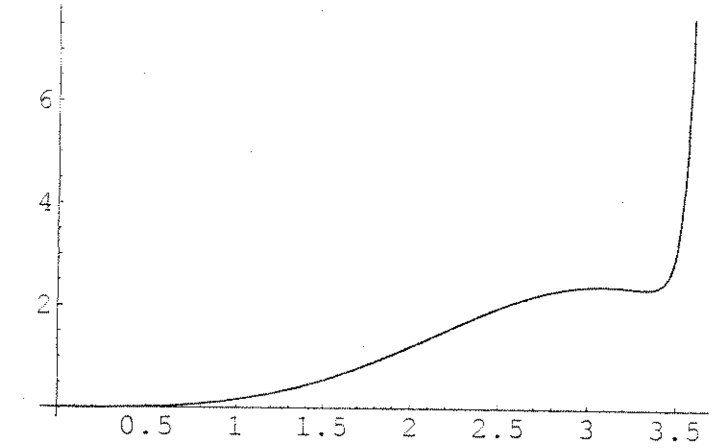

Figure 3:

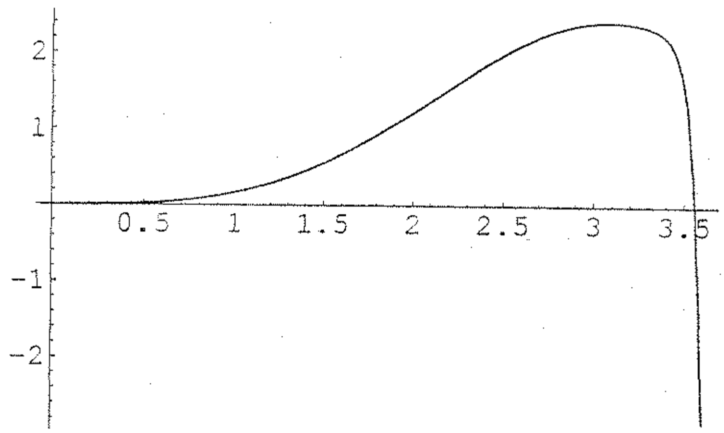

Figure 4:

$$+ 1.0605 \times 10^{-34} x^{63} - 1.55666 \times 10^{-37} x^{68} . \quad (11)$$

The graph of (11) shown in Figure 4. Again the divergence of the series is clear beyond $x = 3.3$.

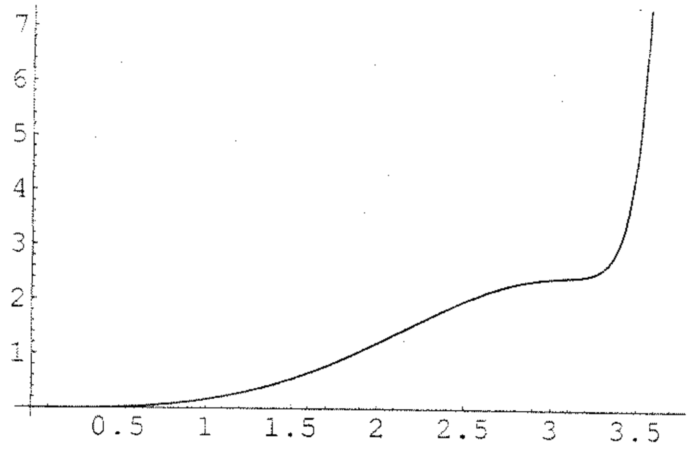

Figure 5: Homotopy solution with $h = 0.1$

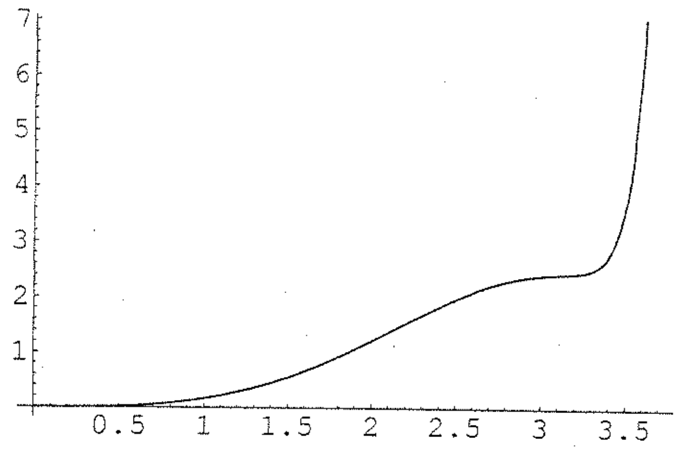

Figure 6: Homotopy solution with $h = -0.1$

5. Conclusion

By applying the Adomian and homotopy analysis methods to our simple nonlinear problem we have shown that these methods produce series solutions which converge over a relatively short interval and it is impossible to get any information for large x. In some cases these methods can yield misleading information regarding the solution of the problem as, for example, the homotopy analysis method in Figures 8-10 gives a zero of the solution whereas the true solution

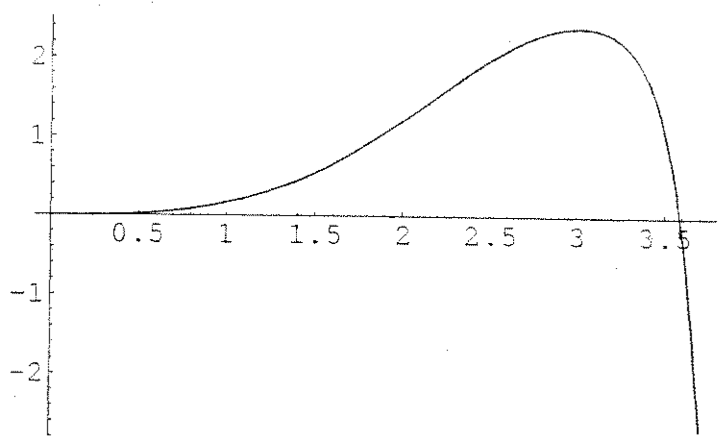

Figure 9: Homotopy solution with $h = 0.5$

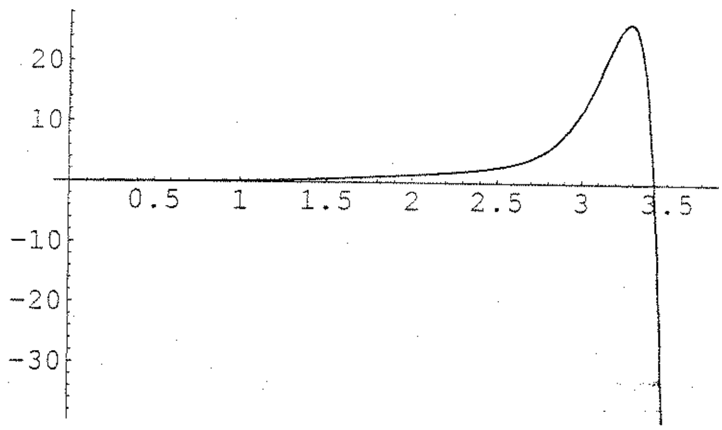

Figure 10: Homotopy solution with $h = -0.5$

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